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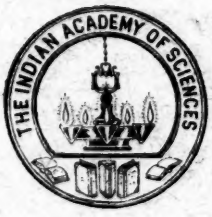

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ON $\sigma_{-1}(n)$ AND $\phi(n)$

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Received January 5, 1943
(Communicated by Prof. B. S. Madhava Rao, D.Sc., F.A.Sc.)

§ 1. Let $\sigma_{-1}(n) = \sum_{d/n} d^{-1}$ and $\phi(n)$ denote the number of numbers less than and prime to n. Further let $M(n)$ stand for the *maximum* order of $\sigma_{-1}(n)$ and $L(n)$ for the *minimum* order of $\phi(n)$. Then Wigert[1] has proved that

$$M(n) \backsim e^{\gamma} \log \log n,$$

and Landau[2] has shown that

$$L(n) \backsim n/e^{\gamma} \log \log n,$$

where γ is Euler's constant.

The object of this paper is to sharpen these results by a different method.

§ 2. Let $F(N) = \prod_{p/n} \left(1 - \frac{1}{p}\right)^{-1}$, where p is a prime; and if $F(N) > F(N')$ for all $N' < N$, let us call N an A and N_r be the rth A.

If $p^2 \mid N_r$, $F(N_r) = F(N_r/p)$, so N_r is not A. Hence N_r is squarefree.

Let $p \mid N_r$ and q be a prime $< p$, which does not divide N_r. Put $N' = q \cdot N_r/p$. Then $N' < N$.

But $\frac{F(N')}{F(N_r)} = \frac{1 - 1/p}{1 - 1/q} > 1$, which is impossible.

So $N_r = 2.3.5 \ldots . p_r$, where p_r is the rth prime.

Let $K(p_r) = F(N_r) - e^{\gamma} \log \log N_r$, (1)

and $N_{r-1} < n \leqslant N_r$.

Then $F(n) \leqslant F(N_r) = F(N_{r-1})/(1 - 1/p_r)$

$$= F(N_{r-1}) + 0\left\{\frac{F(N_{r-1})}{p_r}\right\}$$

$$= e^{\gamma} \log \log N_{r-1} + K(p_{r-1}) + 0(\log \log N_{r-1}/p_r)$$

$$\leqslant e^{\gamma} \log \log n + 0\{K(p_{r-1}) + \log \log n/p_r\}. \quad (2)$$

But when $n = N_r$,

$$F(n) = e^{\gamma} \log \log n + K(p_r). \quad (3)$$

Further $\log N_r = \theta(p_r) \asymp p_r$, which is elementary,

where $\theta(x) = \sum_{p \leqslant x} \log p.$ (4)

Hence, when $N_{r-1} < n \leqslant N_r$, from (2), (3) and (4),

Max. $F(n) = e^\gamma \log\log n + 0\{K(p_r) + K(p_{r-1}) + \log\log n/\log n\}$

But,[2]

$K(p_r) = 0(1)$, (elementary) (5)

$K(p_r) = 0\{1/(\log p_r)^t\} = 0\{1/(\log\log N_r)^t\}$, (deep) (6)

and $K(p_r) = 0\{(\log p_r)^2/\sqrt{p_r}\} = 0\{(\log\log N_r)^2/\sqrt{\log N}\}$, (7)

(on Riemann hypothesis).

Therefore

Max. $F(n) = e^\gamma \log\log n + 0(1)$, (elementary) (8)

Max. $F(n) = e^\gamma \log\log n + 0\{(\log\log n)^{-t}\}$, (deep) (9)

and Max. $F(n) = e^\gamma \log\log n + 0\{(\log\log n)^2/\sqrt{\log n}\}$. (10)

(on Riemann hypothesis).

§ 3. Now we shall consider $L(n)$.

If $b = 0(a)$, then $\frac{1}{a+b} = \frac{1}{a} + 0\left(\frac{b}{a^2}\right)$.

Further $\phi(n) = \frac{n}{F(n)}$ and Min. $\frac{1}{F(n)} = \frac{1}{\text{Max. } F(n)}$.

Hence from § 2, we get

THEOREM I: $L(n) = n/e^\gamma \log\log n + 0\left(\frac{n}{(\log\log n)^2}\right)$, (elementary).

THEOREM II: $L(n) = n/e^\gamma \log\log n + 0\{n/(\log\log n)^t\}$, (deep) for any fixed t.

THEOREM III: $L(n) = n/e^\gamma \log\log n + 0\{n/\sqrt{\log n}\}$, (on Riemann hypothesis).

§ 4. In this section, we shall consider $M(n)$.

If p^a/n and p^{a+1} does not divide n, we have

$$\sigma_{-1}(n) = \frac{\Pi}{p/n}\{(1 - 1/p^{a+1})/(1 - 1/p)\} \leqslant \frac{\Pi}{p/n}\left(1 - \frac{1}{p}\right)^{-1} = F(n) \quad (11)$$

Let $N = 2^{a_2} \cdot 3^{a_3} \ldots . p^{a_p}$,

where $a_\lambda = \left[\frac{\log p}{\log \lambda}\right]$, $[x]$ standing for the integral part of x.

Now $a_\lambda > \dfrac{\log p}{\log \lambda} - 1$. Hence $\log\ \lambda\ (1 + a_\lambda) > \log p$,

so that $\dfrac{1}{\lambda^{a_\lambda+1}} < \dfrac{1}{p}$.

Further when $\sqrt{p} < \lambda \leqslant p$, $a_\lambda = 1$, so that $\lambda^{a_\lambda+1} = \lambda^2$.

Hence if $P = \Pi\,(1 - 1/\lambda^{a_\lambda+1})$, where λ runs through all the primes $\leqslant p$,

$$\log\ P = 0\left\{\sum_\lambda \frac{1}{\lambda^{a_\lambda+1}}\right\} = 0\left\{\sum_{\lambda<\sqrt{p}} 1/\lambda^{a_\lambda+1} + \sum_{\lambda>\sqrt{p}} 1/\lambda^{a_\lambda+1}\right\}$$

$$= 0\left\{\sum_{\lambda<\sqrt{p}} 1/p + \sum_{\lambda<\sqrt{p}} 1/\lambda^2\right\} = 0\,\{S_1 + S_2\}.$$

$$S_1 \leqslant \frac{\pi(\sqrt{p})}{p} = 0\left(\frac{1}{\sqrt{p}\cdot\log p}\right),$$

where $\pi(x)$ denotes the number of primes $\leqslant x$, so that $\pi(x) = 0\,(x/\log x)$.

Now[2] $p_n > kn \log n$

So $S_2 = 0\left\{\sum_{n>c\cdot\frac{\sqrt{p}}{\log p}} 1/n^2\,(\log n)^2\right\} = 0\,(1/\sqrt{p}\cdot\log p)$.

Hence $\log P = 0\,(1/\sqrt{p}\,\log p)$, so that

$$P = e^{0(1/\sqrt{p}\cdot\log p)} = 1 + 0\,(1/\sqrt{p}\,\log p).$$

Therefore, $\sigma_{-1}\,(N) = \underset{\lambda\leqslant p}{\Pi}\,\{(1 - 1/\lambda^{a_\lambda+1})/(1 - 1/\lambda)\}$

$$= F\,(2.3\cdots p)\cdot P = \{e^\gamma \log\left(\theta(p)\right) + K\,(p)\}\,\{1 + 0\,(1/\sqrt{p}\,\log p)\}$$

$$= e^\gamma \log\,\{\theta\,(p)\} + 0\left\{\frac{\log\,\theta\left((p)\right)}{\sqrt{p}\,\log p} + K\ (p)\right\}. \qquad (12)$$

But $\log\ N = \theta\,(p) + \theta\,(p^{\frac{1}{2}}) + \theta\,(p^{\frac{1}{3}}) + \ldots = \theta\,(p) + 0\,(\sqrt{p}\,\log p)$,

so that $\theta\,(p) = \log N + 0\,(\sqrt{\ }\log N \times \log\log N)$

and $\log\,\theta\,(p) = \log\log N + 0\ (\log\log N/\sqrt{\log N})$

Hence $\sigma_{-1}\ (N) = e^\gamma\ \log\log N + 0\left\{\dfrac{\log\log N}{\sqrt{\log N}} + K\,(p)\right\}\Big\}$. (13)

But we have from (11) that

$$\sigma_{-1}\ (n) \leqslant F\,(n). \qquad (14)$$

Therefore from (13), (14), (5) and (8), we get

THEOREM IV: $\sigma_{-1}\ (n) = e^\gamma\ \log\log n + 0\,(1)$, (elementary).

From (13), (14), (6) and (9), we get

THEOREM V: $\sigma_{-r}(n) = e^{\gamma} \log \log n + 0\left\{\frac{1}{(\log \log n)^{r}}\right\}$ (deep).

From (13), (14), and (10), we get

THEOREM VI: On Riemann hypothesis,

$$\sigma_{-1}(n) = e^{\gamma} \log \log n + 0\left\{\frac{(\log \log n)^{2}}{\sqrt{\log n}}\right\}.$$

§ 5. In the papers entitled "Highly Abundant Numbers" and "Totient Numbers", which are unpublished and formed a part of my D.Sc. thesis, I proved the above results. But that proof depends upon the properties of those numbers. This proof is much shorter and independent. Further, the results proved here on assuming Riemann hypothesis are better than my old results by a factor $(\log \log n)^2$.

REFERENCES

1. Wigert .. *Acta Mathematica* for 1914.
2. Landau .. *Primzahlen.*

A NOTE ON VINOGRADOW-NUMBERS

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Received October 28, 1942
(Communicated by Prof. S. Bhagavantam)

Introduction

Let N be any positive integer, $H = e^{\frac{1}{2}\sqrt{\log N}}$,

$$\tau = NH^{-1},\ \tau_1 = NH^{-3}.$$

Then we define, following Vinogradow, a number $\alpha\ (0 < \alpha \leqslant 1)$ to be on a minor arc, if there is an irreducible fraction $\frac{p}{q}$ such that

$$\alpha = \frac{p}{q} + \frac{\theta}{q\tau},\ \text{where } 0 \leqslant \theta \leqslant 1,\ \tau_1 < q \leqslant \tau$$

Let

$$S(\alpha, N) = \sum_{p \leqslant N} e^{2\pi i \alpha p},$$

where p runs through all prime numbers $\leqslant N$. Vinogradow proved in his solution of Goldbach's problem that corresponding to any given $h > 1$ there is a constant $c = c(h)$ such that

$$|S(\alpha, N)| < c\frac{N}{(\log N)^h}, \qquad (1)$$

whenever α is a number inside a minor arc. As N changes, the set of numbers α for which Vinogradow has proved the inequality (1) holds good also changes. From this it does not follow that there are numbers α such that (1) is true for the same α and all sufficiently large N. The real numbers $\alpha\ (0 < \alpha \leqslant 1)$ can be classified into two classes according as the inequality (1) is satisfied or not for all N and the same α.

Definition.—We define α to be a Vinogradow-number or shortly a V.N., if given any constant $h > 1$, there is another constant $c = c(h)$ such that

$$|S(\alpha, N)| < c\frac{N}{(\log N)^h} \text{ for all N.}$$

If a particular number α (mod 1) is always inside a minor arc corresponding to each N, it clearly satisfies a sufficient condition for being a V.N. In this note we prove not only the existence of V.N.'s but also

THEOREM I : The measure of the set of V.N.'s in (0, 1) is unity,

THEOREM II : Every infinite simple continued fraction whose partial quotients a_n satisfy the inequality

$$a_n < e^{\left(\frac{n}{15}\right)^{\frac{1}{2}}}$$

on and after a certain value of n is a V.N.,

THEOREM III : Every irreducible fraction is or is not a V.N. according as its denominator is not or is quadratfrei.

Before taking up the proofs of these theorems it may be straight remarked that $\sqrt{2}$, and in fact every binomial surd, is a V.N. We are not able to give any example of an irrational number which is not a V.N.

Proof of Theorem II.—

Let $\frac{p_n}{q_n}$ and $\frac{p_{n+1}}{q_{n+1}}$ be the nth and the $(n+1)$th convergents to any infinite simple continued fraction of the type mentioned in Theorem II, such that

$$q_n \leqslant \tau < q_{n+1}.$$

When τ is given, there is a unique n with this property.

Now $\tau_1 < q_n \leqslant \tau$,

if $\tau_1 < \frac{1}{a_{n+1}+1}\, q_{n+1}$, $\left(\text{since } \frac{1}{a_{n+1}+1}\, q_{n+1} < q_n \text{ for all } n\right)$,

if $\tau_1 < \frac{1}{a_{n+1}+1}\, \tau$,

if $a_{n+1}+1 < \frac{\tau}{\tau_1} = \mathrm{H}^2$,

if $a_{n+1} < \mathrm{H}^2 - 1$,

if $e^{\sqrt{\frac{n+1}{15}}} < \mathrm{H}^2 - 1$,

if $\sqrt{\frac{n}{15}} < \frac{2}{3}\sqrt{\log \mathrm{N}} + \mathrm{O}(1)$,

if $\frac{n}{15} < \frac{4}{9}\log \mathrm{N} + \mathrm{O}(\sqrt{\log \mathrm{N}})$ (2)

But $q_n > 2 \cdot q_{n-2} > \ldots > 2^{\frac{n-2}{2}}$

$\therefore \log q_n > \frac{n}{2}\log 2 + \mathrm{O}(1)$

$$\therefore \log \tau > \frac{n}{2} \log 2 + O(1)$$

$$\therefore \log N + O(\sqrt{\log N}) > \frac{n}{2} \log 2$$

$$\therefore n < \frac{2}{\log 2} \log N + O(\sqrt{\log N}) \qquad (3)$$

Hence from (2) and (3)

$$\tau_1 < q_n < \tau$$

whenever

$$\frac{2}{15 \log 2} \log N < \frac{4}{9} \log N + O\sqrt{\log N})$$

for all sufficiently large N.

This inequality holds, since

$$\frac{2}{15 \log 2} < \frac{4}{9}.$$

It is well known that

$$\left| \alpha - \frac{p_n}{q_n} \right| < \left| \frac{p_n}{q_n} - \frac{p_{n+1}}{q_{n+1}} \right| = \frac{1}{q_n q_{n+1}} < \frac{1}{q_n \tau}$$

Hence

$$\alpha = \frac{p_n}{q_n} + \frac{\theta}{q_n \tau}, \text{ where } O < \theta < 1 \text{ and } \tau_1 < q_n < \tau.$$

Hence by Vinogradow's theorem quoted above it follows that

$$| S(\alpha, N) | < c \frac{N}{(\log N)^{\frac{1}{4}}}$$

whenever N is sufficiently large. Hence α is a V. N.

Proof of Theorem I.—This follows from Theorem II, and the following

Result of Borel and Bernstein.*—If θ be the class of irrationals in (O, 1), whose partial quotients a_n when expressed as infinite simple continued fractions, satisfy from a certain value of n onwards the inequality

$$a_n < n^{1+\delta}, \ (\delta > 0),$$

then the set is of measure 1.

---

* *Vide* Hardy and Littlewood, "Some problems of Diophantine approximation, Theorem 1·350 along with the explanation, 'The interest of the last theorem lies in the fact that the set in question is of measure one'," *Acta Mathematica*, **37**, 179. Hardy and Littlewood in a footnote in the same page refer to Borel, *Rendiconti di Palermo*, **27**, 247; and *Math. Ann.*, **72**, 578; Bernstein, *Math. Ann.*, **71**, 417.

Proof of Theorem III.—We now require the following lemmas.

Lemma A.—If $(l, q) = 1$, then

$$\pi(N ; q, l) = \sum_{\substack{p \leqslant N \\ p \equiv l \pmod q}} 1$$

$$= \frac{1}{\phi(q)} \int_2^N \frac{dx}{\log x} + O\left(N e^{-\alpha \sqrt{\log N \log\log N}}\right)$$

where α is constant. (This is Satz 402 of Band 2 of Landau's *Zahlentheorie*.)

Lemma B.—If $(a, q) = 1$, then

$$\sum_{\substack{l=1 \\ (l,q)=1}}^{q} e^{2\pi i \frac{a}{q} l} = \mu(q),$$

where $\mu(q)$ is Mobius function. (This is Satz 220 of Band 1 of Landau's *Zahlentheorie*.)

We now proceed to the proof of the theorem.

Let $(a, q) = 1$. Then

$$S\left(\frac{a}{q}, N\right) = \sum_{p \leqslant N} e^{2\pi i \frac{a}{q} p}$$

$$= \sum_{\substack{l=1 \\ (l,q)=1}}^{q} e^{2\pi i \frac{a}{q} l} \pi(N ; q, l) + O(q)$$

$$= \sum_{\substack{l=1 \\ (l,q)=1}}^{q} e^{2\pi i \frac{a}{q} l} \left\{ \frac{1}{\phi(q)} \int_2^N \frac{dx}{\log x} + O\left(N e^{-\alpha\sqrt{\log N \log\log N}}\right) \right\}$$

$$= \frac{\mu(q)}{\phi(q)} \int_2^N \frac{dx}{\log x} + O\left(N e^{-\alpha\sqrt{\log N \log\log N}}\right).$$

The first term on the right side of the above equation vanishes if and only if $\mu(q)$ vanishes. Hence the theorem is proved.

NORMAL CO-ORDINATES OF SYMMETRIC POINT GROUPS : II

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Received January 5, 1943

Introduction.—It has been pointed out in a previous paper in these *Proceedings* that the normal co-ordinates of point groups can be systematically obtained if we know the actual irreducible representations of the point groups. The irreducible representations are known only for very few non-Abelian groups. An alternative method which does not require the knowledge of the irreducible representations has been given in the paper referred to above. In the case of the cubic groups, we give here the actual irreducible representations.

Normal co-ordinates and the irreducible representations of a point group.—Let the symmetry operations of a point system form a group G. It can be easily shown that the several normal co-ordinates of the point system belong to one or other of the irreducible representations of G. For, let the point system consist of n particles and the position of the system be specified by giving any $3n$ independent co-ordinates of the n particles. Let $x_1, y_1 z_1, \ldots . x_n, y_n, z_n$ represent unit vectors through the equilibrium position of the n particles and respectively parallel to a parent system of co-ordinates and let the position vector of any particle r be $a_r x_r + b_r y_r + c_r z_r$ where the co-ordinates of the particle are a_r, b_r, c_r. In the equilibrium position of the molecule a_r, b_r, c_r are all zero. With this notation let the following experssion

$$a_1x_1 + b_1y_1 + c_1z_1 + a_2x_2 + b_2y_2 + c_2z_2 + \ldots . a_nx_n + b_ny_n + c_nz_n,$$

represent the configuration of the system when it is performing a normal mode of oscillation. Let this configuration be represented by Q. Now if we perform a symmetry operation R on the point system we get a new configuration which we may denote by RQ. Obviously the configuration RQ is such that the particles do not necessarily remain in the neighbourhood of their original equilibrium positions. Now if the operation R takes the particle p to the position of the particle q, we shall suppose that the qth particle remains in its own neighbourhood but gets the motion of the pth particle. By doing this for every particle we keep all the particles in the

* One of the papers submitted for the Ramanujam Memorial Prize, 1941.

neighbourhood of their original equilibrium positions and thus obtain a configuration of the system which is geometrically identical with RQ. We may represent this new configuration by $\overline{R}Q$. Obviously the configuration $\overline{R}Q$ must have the same frequency as that of Q. If $\overline{R}Q$ is distinct from Q we obtain two distinct modes of oscillation which have the same frequency. Such a case will not arise for non-degenerate normal modes of oscillation. Now let R run through all the symmetry operations of the group G. For each R of G we find as above the configuration $\overline{R}Q$. All these modes of oscillation may not be linearly independent. Let $Q_1, Q_2, \ldots Q_f$ be a set of linearly independent combinations of several normal modes so that any one of these modes $\overline{R}Q$ is a combination of the f modes $Q_1, Q_2, \ldots Q_f$. Now these f co-ordinates define an irreducible representation of the group G for if R is a symmetry operation

$$\overline{R}Q = a_{11} Q_1 + a_{12} Q_2 + \ldots\ldots a_{1f} Q_f$$
$$\overline{R}Q = a_{21} Q_1 + a_{22} Q_2 + \ldots\ldots a_{2f} Q_f$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\overline{R}Q = a_{f1} Q_1 + a_{f2} Q_2 + \ldots\ldots a_{ff} Q_f$$

Thus $R \rightarrow \overline{R} \rightarrow$ the matrix $|a_{ij}|$

The correspondence is an irreducible representation of G since by our very choice of the f co-ordinates, we cannot find linear combinations of them smaller in number than f which combine among themselves by the transformations defined by all R of G.

The point group T_d.—A point system belonging to this group is a system of particles situated at the four corners of a regular tetrahedron. The symmetry operations for such a system are (1) the identity, (2) eight three-fold rotations about axes passing through the centre of the tetrahedron and any one of the four vertices, (3) three rotations through 180° about the lines joining the middle points of opposite edges (4) six reflections in planes each containing one edge and the centre of the tetrahedron and (5) six rotations through 90° about lines joining the middle points of opposite edges followed by reflections in perpendicular planes through the centre. The group is therefore of order 24 and consists of five conjugate sets of elements. If the particles at the corners be denoted by the numbers 1, 2, 3, 4, the group can be generated by the elements P, Q, R which are respectively the permutations (1234), (123), (12).

The principal axes may be chosen in the following manner. X, Y and Z axes are respectively the lines joining the middle points of the opposite sides 12, 34 ; 13, 24 and 14, 23. x_i y_i z_i denote unit vectors through the particle i (i = 1, 2, 3, 4) and parallel to the co-ordinate system X, Y, Z.

The normal co-ordinates relating to translations and rotations Q_x, Q_y, Q_z and $Q_{\omega x}$ $Q_{\omega y}$ $Q_{\omega z}$ can be immediately written down from their modes and they are

$$\begin{aligned}
Q_x &= x_1 + x_2 + x_3 + x_4 \\
Q_y &= y_1 + y_2 + y_3 + y_4 \\
Q_z &= z_1 + z_2 + z_3 + z_4 \\
Q_{\omega x} &= y_1 - y_2 - y_3 + y_4 - z_1 + z_2 + - z_3 + z_4 \\
Q_{\omega y} &= x_1 - x_2 - x_3 + x_4 - z_1 - z_2 + z_3 + z_4 \\
Q_{\omega z} &= x_1 - x_2 + x_3 - x_4 - y_1 - y_2 + y_3 + y_4.
\end{aligned}$$

Now the equations of transformation are

$$\begin{aligned}
PQ_x &= -Q_z \\
PQ_y &= -Q_y \\
PQ_z &= Q_x
\end{aligned}$$

and therefore P corresponds to the matrix $\begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

Similarly Q and R correspond to the matrices

$$\begin{vmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}$$

respectively. The correspondence is easily verified to be an irreducible representation and is F_2 in the following table. Similarly the three rotations $Q_{\omega x}$, $Q_{\omega y}$, $Q_{\omega z}$ define the irreducible representation F_1. The only representation of 24 as the sum of 5 squares of integers is

$$24 = 3^2 + 3^2 + 2^2 + 1^2 + 1^2$$

and this shows that the group has two one dimensional, one two dimensional and two three dimensional irreducible representations. The one dimensional irreducible representations are directly obtained from the correspondences

$$P \sim 1,\ Q \sim 1,\ R \sim 1$$

and $$P \sim -1,\ Q \sim 1,\ R \sim -1.$$

The two dimensional representation E is now obtained from the orthogonality relations between the irreducible representations. It may be noted that the irreducible representations are unique but for equivalence. In the following table the representative matrices of the basis elements only are given in the several irreducible representations. With the usual rule for the multiplication of matrices, the matrix representation of any other element of the group can be obtained from the matrix representations of the basis elements.

Table of irreducible representations of the point group T_d.

	P	Q	R
A_1	1	1	1
A_2	−1	1	−1
E	$\begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix}$	$\begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix}$	$\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix}$
F_1	$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{vmatrix}$	$\begin{vmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$	$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}$
F_2	$\begin{vmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$	$\begin{vmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$	$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}$

The other cubic point groups.—The point group O is simply isomorphic with T_d and therefore it has the same irreducible representations as for T_d. The irreducible representations of T_d are also the irreducible representations of T. In the case of T, A_1 and A_2 become identical, so also F_1 and F_2. E becomes only a real irreducible representation. The point groups O_h and T_h are the direct products $O \times H$ and $T \times H$ where H is a group of order two. If Γ is an irreducible representation of O (or T_h) the irreducible representations of O_h (or T_h) are obtained by making the basis element of H to represent the matrices I and −I successively where I is the Unit matrix in Γ.

My grateful thanks are due to Prof. S. Bhagavantam for the constant help and encouragement he has been giving me.

THE CHARACTER TABLE OF A SUBGROUP OF THE SYMMETRIC GROUP OF DEGREE 8*

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Received January 5, 1943

Introduction :—Point groups are formed by taking the symmetry operations (1) rotations about symmetry axes, (2) reflections in symmetry planes and (3) rotation reflections. It is possible to extend point groups by introducing other kinds of symmetry operations. The application of such extended groups to some physical problems will be reserved for a later communication. The aim of the present paper is to extend the point group S_8 by introducing a twist about the symmetry axis and to give the character table of this extended group. The abstract form of this group is

$$P^8 = Q^2 = R^4 = E\ ;\quad PQ = QP^{-1}\ ;\quad RP = P^3R^3\ ;\quad RQ = QR^3.$$

It may be considered to be generated by P,Q,R where P,Q,R are respectively the permutations

(15263748), (24) (57), (1234).

A general method of calculating the characters of any group based on Young Tableaux is given in Littlewood's recent book on group characters. Burnside has also given another general method in his book on the theory of groups. But both these methods are only of theoretical importance and they are most unsuitable for use in any particular case. Recently Richardson and D.E. Littlewood have calculated the tables of characters of the symmetric groups up to degree 13. Later Littlewood showed how to calculate the characters of subgroups of symmetric groups. Since any group may be regarded as a subgroup of a suitable symmetry group, his methods are somewhat more practicable. They are particularly useful for subgroups of high order. The group characters can be more systemetically obtained if the following theorem could be proved as suggested by Littlewood.

"If G is a subgroup of H and G has two linear invariants in an irreducible representation of H, then there is a subgroup G of H containing G which has only one linear invariant."

---

* One of the papers submitted for the Ramanujam Memorial Prize, 1941.

Orthogonal relations :—The following very important orthogonal relations between the characters of a group are well known and they are constantly used in obtaining the character table:—

(1) There are as many irreducible representations of a group as there are conjugate classes of elements in it.

(2) The character of the identity element is equal to the dimension of the irreducible representation and is always a divisor of the order of the group.

(3) If x_ρ^λ represents the character of any element S belonging to a class ρ in the irreducible representation λ and if h_ρ is the number of elements in the class ρ

$$(a)\quad \Sigma\, \chi_\rho^{(\lambda)}\, \chi_{\rho'}^{(\lambda)} = 0 \qquad \rho \neq \rho'$$

$$(b)\quad h_\rho \Sigma_\lambda\, \chi_\rho^{(\lambda)2} = h \qquad \text{(the order of the group)}$$

(4)

$$(a)\quad \Sigma_\rho\, h_\rho\, \chi_\rho^{(\lambda)}\, \chi_\rho^{(\mu)} = 0 \qquad \lambda \neq \mu$$

$$(b)\quad \Sigma_\rho\, h_\rho\, \chi_\rho^{(\lambda)2} = h.$$

The character table for the extended group

Class ..	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}
Order ..	1	4	2	4	8	8	8	2	4	2	4	4	1	4	4	4
A_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A_2	1	−1	1	−1	1	1	−1	1	−1	1	1	−1	1	1	−1	−1
A_3	1	1	1	−1	−1	1	−1	1	1	1	−1	1	1	−1	1	−1
A_4	1	−1	1	1	−1	1	1	1	−1	1	−1	−1	1	−1	−1	1
B_1	1	1	1	1	1	−1	−1	1	−1	1	−1	1	1	−1	−1	1
B_2	1	−1	1	−1	1	−1	1	1	1	1	−1	−1	1	−1	1	−1
B_3	1	1	1	−1	−1	−1	1	1	−1	1	1	1	1	1	−1	−1
B_4	1	−1	1	1	−1	−1	−1	1	1	1	1	−1	1	1	1	1
E_1	2	0	−2	0	0	0	0	2	2	−2	0	0	2	0	−2	0
E_2	2	0	−2	0	0	0	0	2	−2	−2	0	0	2	0	2	0
E_3	2	0	−2	0	0	0	0	−2	0	2	2	0	2	−2	0	0
E_4	2	0	−2	0	0	0	0	−2	0	2	−2	0	2	2	0	0
E_5	2	2	2	0	0	0	0	−2	0	−2	0	−2	2	0	0	0
E_6	2	−2	2	0	0	0	0	−2	0	−2	0	2	2	0	0	0
G_1	4	0	0	2	0	0	0	0	0	0	0	0	−4	0	0	−2
G_2	4	0	0	−2	0	0	0	0	0	0	0	0	−4	0	0	2

Extension of the point group S_8 :—An example of a point system coming under the group S_8 is a set of 8 particles of the same kind which occupy the corners of a puckered octagon made up of two equal squares one turned through 45° with respect to the other. The planes of the squares are parellel but not coincident. If the particles at the corners of the squares be represented by the numbers 1, 2, 3, 4 in the lower square and 5, 6, 7, 8 in the upper square, the point group which is of order 16 is easily seen to

be generated by the two permutations (15263748) and (24) (57) (68). A new symmetry operation, namely a twist about the symmetry axis, may be performed as follows.† Keep one of the squares, say the square containing the particles 5, 6, 7, 8 stationary and rotate the other square containing the particles 1, 2, 3, 4 about the symmetry axis through a right angle. The operation may be represented by the permutation R = (1234). The group generated by the three operations P, Q and R is of order 64 and the elements of this group fall into 16 conjugate classes. They are

$C_1 = E$

$C_2 = (R, RP^6, R^3P^2, R^3)$

$C_3 = (R^2, R^2P^4)$

$C_4 = (RQP^2, RQP^6, R^3QP^2, R^3QP^6)$

$C_5 = (Q, QP^2, QP^4, QP^6, R^2Q, R^2QP^2, R^2QP^4, R^2QP^6)$

$C_6 = (P, P^3, P^5, P^7, R^2P, R^2P^3, R^2P^5, R^2P^7)$

$C_7 = (RQP, RQP^3, RQP^5, RQP^7, R^3QP, R^3QP^3, R^3QP^5, R^3QP^7)$

$C_8 = (P^2, P^6)$

$C_9 = (RP, RP^5, R^3P^3, R^3P^7)$

$C_{10} = (R^2P^2, R^2P^6)$

$C_{11} = (R^2QP, R^2QP^3, R^2QP^5, R^2QP^7)$

$C_{12} = (RP^2, RP^4, R^3P^4, R^3P^6)$

$C_{13} = P^4$

$C_{14} = (QP, QP^3, QP^5, QP^7)$

$C_{15} = (RP^3, RP^7, R^3P, R^3P^5)$

$C_{16} = (RQ, R^3Q, RQP^4, R^3QP^4)$

Since there are 16 conjugate classes of elements in the group, there will be 16 irreducible representations for this group. The following eight correspondences obviously define eight one dimensional irreducible representations

$$P \sim \pm 1, Q \sim \pm 1 \text{ and } R \sim \pm 1.$$

Since all the possible combinations are taken, there can be no other one dimensional irreducible representation. Now the only way of expressing 64 which is the order of the group as the sum of 16 squares of integers of which eight are equal to unity is the following one

$$64 = 8 \cdot 1^2 + 6 \cdot 2^2 + 2 \cdot 4^2.$$

† In the practical case of sulphur, which conforms to the above model, this new operation does not exist because there are chemical bonds connecting 15, 25, 26, etc.

This relation shows that the group has 8 one dimensional and 6 two dimensional and 2 four dimensional irreducible representations. Since the character of the identity element is equal to the dimension of the corresponding irreducible representation, the first column in the character table is at once written. The correspondence

$$P \sim \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \text{ and } Q \sim \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

defines a two dimensional irreducible representation of S_8. Now if we make R to correspond to the two matrices.

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad \text{and} \quad \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$$

we obtain the irreducible representations E_1 and E_2 and the characters in the two irreducible representations are now written. From the orthogonality relations it follows that the remaining elements in the columns 2, 3, 5, 6, 7 are all zero. Following Littlewood's method, the other elements in the character table are obtained after a few trials from the compound characters derived from the characters of the symmetric group of degree 8. It may be mentioned that Littlewood has given the character tables of the subgroups of order 1344, 1152, 192, 384 and 336 of the symmetric group of degree 8. The character table of the group of order 64 considered in this paper has not been previously published. An effective check for the correctness or otherwise of the character table is finally made with the help of the orthogonality relations of the group characters.

My grateful thanks are due to Prof. S. Bhagavantam for the constant help and encouragement he has been giving me.

ATMOSPHERICS DURING THE MONSOON PERIOD

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Introduction

The present paper is a study of Atmospherics in relation to meteorological depression in the Bay of Bengal. The work was carried out in the Physics Laboratory of the Annamalai University, in the years 1940 and 1941, during the period of the summer monsoon.

The relation of atmospherics to weather has been studied by Jackson, Loring, Delval, De Groot, Bureau and others. Jackson (1902) found that atmospherics were more frequent in summer than in winter, and in front of a cyclonic depression rather than in the rear. The work of Loring (1911), Delval (1914) and De Groot (1917) showed that the monsoon was often accompanied by heavy thunderstorms and atmospherics, and that prolonged cracklings indicated stormy weather. Bureau and others (1924) found that atmospherics always appear from regions over which a mass of fresh polar

air was passing, and that a more marked relation could be traced in winter as opposed to summer. The Committee of the Royal Meteorological Society on Atmospherics and Weather (1927) found that many of the sources of atmospherics lay in regions of meteorological disturbance. Nasilov (1937) pointed out that the interference by atmospherics is caused in the lower strata of the atmosphere and is therefore dependent on a number of variable factors.

With regard to the relation between atmospherics and lightning Mosler (1911), Goldschmidt (1920) and Lugeon (1925) found that atmospherics originating from lightning in the lower layers of the atmosphere have a range not exceeding 250 km. Boswell (1932) showed that automatic recorders could be used to trace atmospheric sources and therefore thunderstorms up to a distance of 1500 miles. The records of atmospherics obtained by Leithauser (1936) showed that the sources of atmospherics varied in number throughout the day, being a few in the morning, and rising to a maximum in the afternoon and at sunset.

The Radio Telegraphic Investigations Committee of the British Association in their investigation on atmospherics found that atmospheric storms coincided with convective weather, rapid fluctuations of pressure and movement of depressions. Lacoste (1921) showed that atmospherics are due to more or less distant storm phenomena, are often produced in overcast regions ; and that the course of depressions could be followed with the aid of atmospherics accompanying a well-formed depression. Wolf (1922) found that during the warm half-year, atmospherics developed on the appearance of a new depression and disappeared on its departure or filling up, and that no such relation could be traced in winter. Paoloni (1938) found a close relation between atmospherics and barometric pressure.

Though there is general agreement on the main correlation of atmospherics and meteorological data, there is a large divergence of opinion regarding the details. That there is such a divergence of opinion is no wonder, when we remember that a generally acceptable, unambiguous scale and classification of atmospherics has not yet been evolved. Further, the nature of the equipment used by various observers differ widely. As Napier Shaw (1932) says, the combination of recording and direction-finding of atmospherics, and the location of thunderstorms is a subject which the meteorological laboratoty has yet to solve.

Experiment

General plan of the investigation.

For the study of atmospherics in relation to meteorological conditions, records were taken daily throughout the day during specific periods of the

year. With the aid of these records it is possible to follow not only the diurnal variations of atmospheric activity, but also its variation from day to day.

First, an attempt was made to simplify the apparatus as much as possible. In the preliminary investigations carried out, by the author (1935), it was found that records for a period of two minutes were sufficiently representative of the atmospheric activity at the time. Further, as pointed out by the author (1938), the records give valuable information regarding the gross structure of atmospherics.

Finally a Pye 803 5-valve superhet and a moving coil galvanometer were employed for the purpose. In place of continuous recording, it was decided that sample records of atmospherics for about two minutes be taken every hour throughout the day. The arrangement was rendered automatic by a special device constructed in the laboratory, details of which were given by me in a previous paper (1941).

Apparatus.

The apparatus used in the present investigation consists of two Pye 803 5-valve superhets with the A. V. C. circuits put out of action. One of the sets was tuned to a wavelength of 375 metres and the other to 37·5 metres. The outputs of these receivers were suitably connected to two moving coil galvanometers of period 2 seconds. The spots of light from the mirrors of the two galvanometers were focussed on a rotating drum and their movement recorded on bromide paper. Records were taken for about two minutes at intervals of one hour throughout the day. With the aid of an automatic switching device, described by the author and Subrahmanyam (1938), the two receivers, the galvanometers and the motor used for rotating the drum were switched on at intervals of one hour. Atmospherics were then recorded for about 2 minutes and the entire apparatus was automatically switched off.

Graphical representation of "Atmospheric Activity".

Estimates of the intensity of atmospherics were made from the records on a scale of 5. As pointed out in an earlier (1935) paper the kick of the galvanometer has been taken as a measure of the intensity of the atmospheric or atmospheric group reaching the observer. The number of atmospherics received in one second could also be easily counted. When the atmospherics come in groups in quick succession (in less than half a second) it is not possible to estimate the strengths of the individual components of the group and in such cases the throw of the galvanometer has been taken as representing the intensity of the group.

The sum of all the impulses received by each galvanometer during one minute has been taken as the measure of " Atmospheric Activity " and this quantity has been plotted along the Y-axis in the various graphs. The ordinates indicate the times of the day at which the observations are made.

Although records were taken on two wavelengths, it was very soon found that the records on the short waves were unreliable at certain times. On account of the congestion of traffic now prevailing on the short waves on account of the war, it was found exceedingly difficult to keep the receiver tuned to a wavelength in the short-wave bands, which was perfectly silent, except for atmospherics, throughout the day. In the present paper, therefore, only atmospherics on the medium waves are discussed.

Results

Diurnal variation of atmospherics.

In an earlier paper by the author (1935) it was pointed out that in general the number of atmospherics received per minute shows a maximum either between 2 P.M. and 4 P.M. or between 6 P.M. and 8 P.M. A second maximum sometimes appears between midnight and 4 A.M. Those curves if re-plotted with " atmospheric activity " substituted for the number of atmospherics received per minute, bear out the same general features.

It was further pointed out that the periods of maximum atmospheric activity appear to fall into three divisions : (1) afternoon at about 4 P.M., (2) early night 8–10 P.M. and (3) early morning at about 4 A.M. and that these seem to indicate the origin of these atmospherics. " The observations made by the Challenger show that the frequency of thunderstorms at sea have a pronounced maximum between 2 and 4 A.M. while the occurrence of lightning shows a maximum between 8 and 10 P.M. Over land, heat thunderstorms occur most frequently during the afternoons. But there are certain types associated with the line of separation between warm and cold air currents which show no special preference to day or night." The present investigations seem to support the above observations in a very large measure.

Atmospherics during the monsoon period.

In the present paper it is proposed to discuss the nature of the atmospheric activity curves in relation to the meteorological data collected at a large number of places distributed throughout India and published daily by the Indian Meteorological Department, as the *Indian Daily Weather Report*. The period chosen for study in the present paper includes the months of August and September in the years 1940 and 1941. This period

is described as the period of the Summer Monsoon. During this period the Peninsula and several parts of India receive copious rainfall. Most of the rainfall is due to the monsoon which again is controlled by storms and depressions in the Bay of Bengal and the Arabian Sea.

During the period under discussion depressions usually form in the Bay of Bengal and gradually move either west or north-west into Central India and finally disappear from North-West India. Annamalainagar where the University laboratories are located and from which the investigation has been carried out is situated in Lat. 11°–25′ N., Long. 79°–44′ E., on the East Coast of India. The location of the present observation post is indeed fortunate in that it is neither too near, nor too far away from the places of origin and the usual tracks of the meteorological depressions and cyclones which cross the coast of the Bay of Bengal frequently during this period.

August 1940.—The chief feature of the month was a steady monsoon with well-distributed rainfall. The activity of the monsoon was mainly controlled by one storm and three depressions from the Bay of Bengal. The cyclonic storm which formed at the head of the Bay of Bengal on the 2nd, lay as a depression over Chota Nagpur on the 4th, to become unimportant by the 11th. The first of the depressions formed off the Orissa Coast on the 12th and moving westwards to West Central India filled up on the 16th. The next depression formed off the Orissa Coast on the 21st and travelling across the country disappeared on the 27th. Both these depressions caused active monsoon conditions in the Peninsula, central parts of the country and North-West India between the 11th and the 26th.

This period of one fortnight in August, provided an excellent opportunity for the present investigation and the atmospherics during this period will be discussed in detail later on.

September 1940.—This month did not provide good examples of depressions in the Bay of Bengal. A depression formed in the Bay of Bengal on the 15th and crossing inland on the 17th lay as a depression on the 18th. Another depression formed in the North Andamans sea, moved towards the head of the Bay of Bengal on the 21st and filled up there the following day. The last week of the month noticed the withdrawal of the monsoon from the country (from *Indian Weather Review*, 1940. *Monthly Weather Reports*, for August and September, 1940).

August 1941.—During this month there was no well-defined depression in the Bay of Bengal. There was a break in the monsoon in the last third of the month. There was marked thunderstorm activity in the south and east of the Peninsula towards the end of the month.

September 1941.—There were two depressions during this period but neither was well defined. The second moved from the Bay of Bengal across the Deccan into North Bihar. This caused good rainfall in most parts of the Peninsula and in the West Central Provinces between the 13th and 19th. The monsoon withdrew from North-West India by the 20th and from the central parts of the country by the 22nd. (From *Monsoon Rainfall Summary*, June-September 1941. Supplement to the *Indian Daily Weather Report* for 14th October 1941.)

Atmospherics accompanying " unsettled conditions " in the Bay of Bengal.

The atmospheric activity curves which exhibit a particular characteristic are grouped together and presented. On certain days the atmospheric activity is not confined to particular periods but exists throughout the night. Variation in activity from hour to hour is noticed but the variation is such as to indicate beyond doubt the presence of atmospheric activity throughout the night. Eight such curves are given in Fig. 1 and Fig. 2. These curves fall into groups. Each group consists of one, two or in some cases three curves. Each curve covers a period of 24 hours, commencing from 12 hours on a particular day and ending at 12 hours on the following day. The ordinates corresponding to midnight are indicated by means of a line parallel to the Y-axis in the figures. This method of plotting the curves is adopted for two reasons. First, on most of the days there is very little activity round-about midday. Activity commences usually after sunset and continues with varying vigour through the dark hours and on several occasions suddenly diminishes with the dawn. Again in the morning hours till midday there is rarely any activity. Hence to show clearly the continuity of atmospheric activity through the night the method of plotting from midday to midday is found to be advantageous. Secondly, this method of plotting brings the more important parts of the curve into the centre of the field.

The details of the various curves in Figs. 1 and 2 are given below.

The last column gives extracts from the *Indian Daily Weather Reports* which are issued every day and are made up of observations recorded at 8 A.M. at a number of places distributed throughout the country. The dates of the weather reports correspond to the dates given in the previous column.

It should be mentioned in this connection that all the curves are not drawn to the same scale. The scale on the X-axis is the same for all the curves. On the Y-axis, however, there are two scales. In curves 1*a* and 2*a* one unit of length on the Y-axis represents double the amount of atmospheric activity in the other curves. On these days atmospheric activity was very

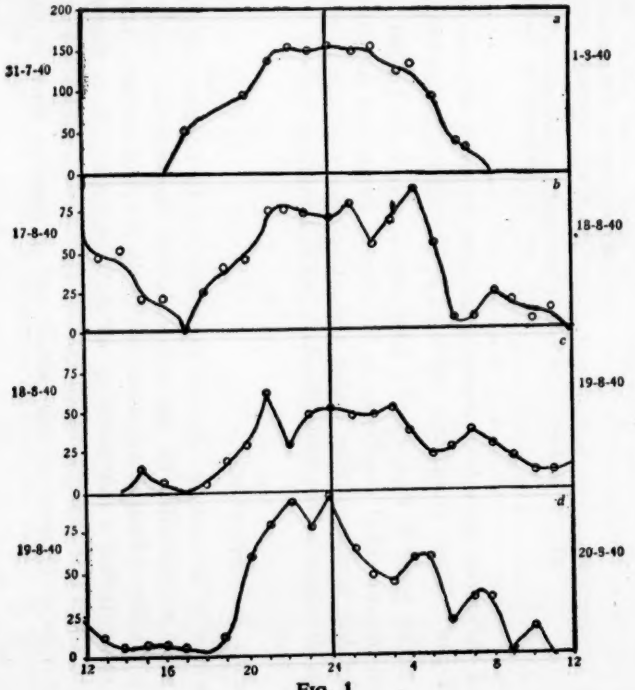
31-7-40
1-9-40
17-8-40
18-8-40
18-8-40
19-8-40
19-8-40
20-9-40
a
b
c
d
200
150
100
50
0
75
25
12
16
20
24
4
8
12

FIG. 1

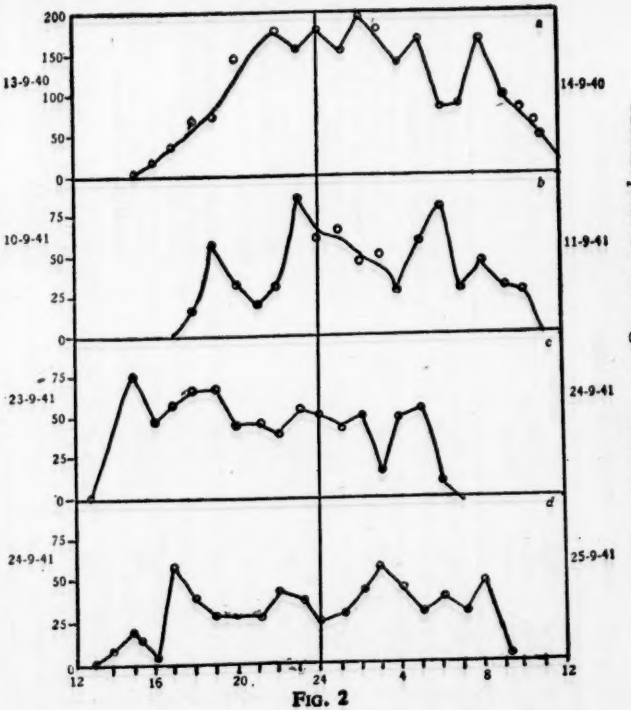
13-9-40
14-9-40
10-9-41
11-9-41
23-9-41
24-9-41
24-9-41
25-9-41
a
b
c
d
200
150
100
50
0
75
25
12
16
20
24
4
8
12

FIG. 2

TABLE I

Group	No. of curves in group	Serial No. of curve	Dates		Extracts from the *Indian Daily Weather Reports*
			From	To	
A	1	1*a*	31-7-40	1-8-40	Conditions are unsettled at the head of the Bay of Bengal.
B	3	1*b*	17-8-40	18-8-40	The low over east C.P. and Orissa has extended into the Bay of Bengal off the Circars coast where weather is markedly unsettled.
		1*c*	18-8-40	19-8-40	Conditions continue unsettled off the Orissa coast where a depression may form.
		1*d*	19-8-40	20-8-40	Conditions are unsettled in the east-central Bay of Bengal where a depression may yet form.
C	1	2*a*	13-9-40	14-9-40	Weather is unsettled over the north and central Bay of Bengal.
D	1	2*b*	10-9-41	11-9-41	Conditions are unsettled in the east-central Bay of Bengal and the adjoining parts of the North Andamans Sea.
E	2	2*c*	23-9-41	24-9-41	A trough of low pressure lies this morning off the Chittagong-Arakan coast where a depression may form.
		2*d*	24-9-41	25-9-41	The unsettled conditions in the north-east Bay of Bengal are less marked.

great and the curves have been rendered uniform by using different scales on the Y-axis. These are clearly indicated in the figures themselves.

The curves given in Figs. 1 and 2 taken along with Table I point to the conclusion that unsettled conditions in the Bay of Bengal are accompanied by atmospheric activity curves which indicate almost continuous activity throughout the night. Why there should be such continuous activity will be discussed later.

Atmospherics and depressions.

Formation of depressions.—It will be interesting to observe next, what characteristics are exhibited on the days following the unsettled conditions reported in the Bay of Bengal, and described in the previous section. Limitations of space forbid a discussion of all available cases. It is therefore, proposed to select one example which exhibits clearly what appear to be the important typical features of this period and to describe them in detail.

Other examples will then be briefly cited to lend additional support to the conclusions that become evident. The example so chosen refers to the period, 20th to 23rd August 1940.

20th-23rd August 1940.—Two curves *a* and *b* relating to this period are given in Fig. 3. Curve 3*a* refers to the period from 12 hours on the 20th August to 12 hours on the 21st August. It shows two distinct peaks, one at 19 hours on the 20th and the other at 5 hours on the 21st. It was pointed out by the author (1941) in a paper on the " Effect of Sunset on Atmospherics " that a classification of atmospherics according to intensities on this day at 19 hours suggests the existence of a previous state of preparation in the atmosphere for the onset of a thunderstorm. It is further pointed out that on those days on which the trigger action of sunset is most pronounced, the peak activity is reached very early in the night (at about 20 hours), whereas on those days on which the trigger action is not so pronounced, the rate of growth of thunderstorm activity is slow and the peak activity is attained very much later in the night between 10 P.M. and midnight. We may therefore attribute the peak A to the activity initiated by sunset. Peak B which is quite distinct from A may be due to activity at a place where, on account of the unprepared state of the atmosphere, the maximum activity occurs a little later on. Thus peaks A and B may be attributed to activity over land, the atmospherics being generated at different centres at different times.

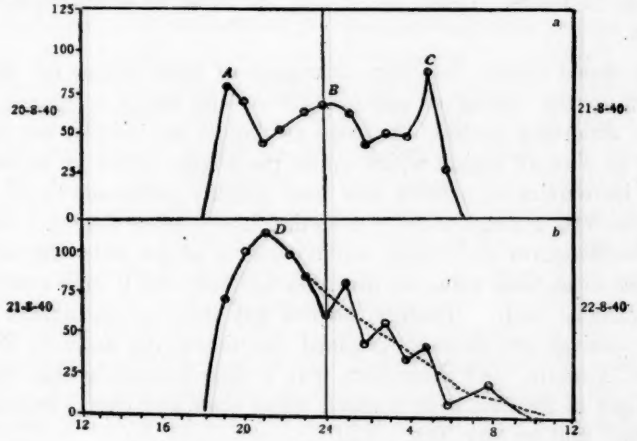

FIG. 3

The third peak C which occurs in the early morning is of special interest. According to the observations made by the " Challenger ", thunderstorms

at sea have a pronounced maximum between 2 A.M. and 4 A.M. The peak C may therefore be due to activity over the sea. It must also be noted that the peak C is quite sharp unlike B. This indicates that the phenomena which gives rise to the atmospherics observed during this period come from a single narrow region. There is a sudden rise to peak C and then again a sudden fall from 89 at 5 hours to 28 at 6 hours, and to a zero value at 7 hours.

Curve 3*b* refers to the period from midday of the 21st to midday on the 22nd. This curve shows characteristics which are in some ways similar to those exhibited by the previous curve and in some ways entirely distinct. The initiation of activity in the evening of the 21st is by the trigger action of sunset. (This case also was discussed in detail by me in the paper on "The Effect of Sunset on Atmospherics".) The peak activity D (112) is very much greater than that represented by A (80) on the previous day.

To explain this difference between the two values it may be suggested that the sources of atmospheric activity may be nearer the observing station on the second day. A detailed examination of the available data shows, however, that such an explanation is not plausible. Let us compare the activities at different hours between sunset and midnight on the 20th and 21st. Starting practically with zero activity at 18 hours on both the days, the activity rose up to a maximum of 80 at 19 hours on the 20th and fell to 45 by 21 hours. On the 21st however, the activity was only 69 at 19 hours. But the increase in activity still continued and a maximum of 112 was reached at 21 hours. Later, the activity fell down to 45 only at 2 hours on the 22nd.

This shows clearly that the occurrence of large values on the 21st do not point to the centres of atmospheric activity being at shorter distances from the observing station. It looks as though the activity on the 20th is confined to a small region which under the trigger action of sunset quickly reached its maximum activity and also quickly exhausted itself, whereas, that on the 21st appears to have extended into a wider region. The growth of the thunderstorm to its peak activity took a longer time, the activity was maintained at its peak value for nearly three hours and it took a much longer time to exhaust itself. Further, on this day the average number of atmospherics received per minute remained practically the same at 29 from 20 hours to 23 hours. All these facts give a clear indication that the activity on the night of the 21st is on a much vaster scale and over a more extensive region than that on the 20th.

While the atmospheric activity before midnight has shown a distinct increase over that of the previous day, we find that activity after midnight

does not show any peak like that at C. The undulating curves in 3*b* show a general tendency to a gradual decay of atmospheric activity. This feature may be more clearly brought about by drawing the dotted curve passing through the mean values instead of through the points representing the observed values. This is quite justifiable as the variations from the mean are not large.

The atmospheric activity from the 21st to the 22nd as represented by curve 3*b* may now be briefly summarised. The curve is the result of thunderstorm activity over land in a prepared atmosphere, the activity being initiated by sunset and lasting till 23 hours at a large value, and later on slowly decreasing and attaining a zero value only by about 10 hours on the 22nd. There is no activity of any distinct character in the early hours of the morning.

The Weather Reports.—It will now be very interesting to turn our attention to the weather reports for this period. The weather during the 24 hours ending with 8 hours on the 21st is described as follows : "The unsettled conditions off the Orissa Coast developed into a depression centred this morning about 100 miles east-south-east of Puri." It will be remembered that during the three days preceding this, the weather off the Circars Coast has been described as markedly unsettled. The curves for these three days are given in Figs. 1*b*, 1*c* and 1*d* and the details regarding these curves are given in Table I. The formation of a depression was expected even on the morning of the 19th but it actually occurred on the 21st. Again on the 22nd it is reported that "the depression has passed inland" and that it "lies at a distance of 50 miles north-east of Cuttack".

For the purpose of a more detailed study, the weather charts published on the 21st and the 22nd are redrawn in Fig. 4. The isobars refer to 8 A.M. on the dates mentioned. It will be noticed that in addition to the isobars a number of black dots are marked in the figures. In the *Indian Daily Weather Reports*, data relating to the occurrence of thunderstorms during the previous 24 hours are also included. The position of the observing stations from which thunderstorms are reported on these days have been indicated by the black dots. It should not however, be assumed that the thunderstorms are confined to only those areas covered by the dots. In addition to these, it will be noticed that three dotted concentric circles are drawn. The centre of these circles is at Annamalainagar. The smallest circle indicates points which are at a distance of 250 miles from Annamalainagar, the second those at a distance of 500 miles and the third those at a distance of 750 miles. These circles enable us to judge approximate distances of points from the centre.

Comparing Figs. 4*a* and 4*b*, we notice immediately that the centre of the depression has shifted, as reported in the *Indian Daily Weather Report*, from a position on the sea to a point on the coast. Next we notice that the distribution of areas from which thunderstorms are reported show a marked change. In Fig. 4*a* there are at least two regions along the coast from which thunderstorm activity is reported. They are at a distance of about 500 miles from the apparatus employed in the present investigation. In addition there is a point *a* which is at a distance of 250 miles. It is situated inland and almost at the centre of the Peninsula. There are three others *b*, *e* and *f* situated inland and at a distance of 500 miles. There are also several other points which are at still greater distances. It must also be remembered that the regions of activity might have extended into the sea but as there are no observing stations on the sea, they cannot be marked in the figure. But it would be quite reasonable to assume that disturbances reported from points *c* and *d* define only the outskirts of the active region, the regoin itself extending seaward.

In Fig. 4*b* there are practically no points along the coast except for one which is at a distance of 750 miles. The other points have all moved inland and farther away from the coast.

The movement inland of the areas from which thunderstorms are reported during the previous 24 hours and the disappearance of thunderstorm activity at points along the coast seems to coincide with the appearance of peak C in Fig. 3 in the early morning of the 21st and its complete disappearance on the morning of the 22nd. It may therefore be suggested that the occurrence of large atmospheric activity in the early hours of the morning, leading to a well-defined peak like C, seems to point to the formation of a depression in the Bay of Bengal. The movement of the depression inland is then associated with increased early night activity initiated by sunset, while the activity in the early hours of the the morning disappears. With the depression moving still further inland the scale of the phenomena becomes very much reduced.

Sources of atmospherics actually recorded.—The atmospherics recorded might have been those generated at any one of the places marked in the figures, but one can never be quite certain which place they come from. Whether a particular atmospheric generated at a particular place can affect the receiver at the observing station depends on the strength of the atmospheric, its distance from the receiver, and the sensitivity of the receiver and recording equipment. It may be assumed that the equipment used in the present investigation is capable of responding to a very powerful thunderstorm raging at a distance of about 750 miles.

It is sometimes found that on certain days thunderstorm activity is not reported from places within short distances from the receiver, but still atmospherics of sufficient intensity are recorded. The question may then be asked, wherefrom do the recorded atmospherics arise? The weather reports mention observations from places where thunder is heard, with or without precipitation, and therefore relate only to those phenomena that take place in the comparatively lower layers of the atmosphere. But lightning discharges might occur at higher levels, invisible from the surface and unaccompanied by thunder. Some of the recorded atmospherics may arise in the upper atmosphere over regions of meteorological disturbance sometime before such discharges become visible to ground observers. It may thus be possible in certain cases, to predict the formation of depressions with the aid of atmospherics in advance of the weather reports.

The above is only one of eight other examples of atmospheric activity, which bear out the general characteristics mentioned above.

Nature of the depression indicated by atmospheric activity.—A detailed consideration of atmospheric activity curves between the 13th and 18th September 1940 shows some more interesting features. In Fig. 5 are given three atmospheric activity curves. Curve 2*a* may be taken along with these for purpose of this discussion. The curves 2*a* and 5*a* which refer to the period 13th to 16th, show atmospheric activity throughout the night. Curve 5*b* is different from the previous curves in that the peak activity is reached at 22 hours on the night of the 16th, and once again in the early morning of the 17th. Again Curve 5*c* is different from 5*b*. It shows a small peak at 19 hours and then a large peak at 22 hours. From that time onwards, there is a rapid fall of activity till 4 hours and then a small rise at 5 hours followed by zero activity at 6 hours. The curve thus shows a very large activity before midnight and comparatively small activity after midnight. In this respect it differs from the previous curve, in which activity after midnight is either practically the same or slightly greater than the activity before midnight.

Summary of the Weather Reports during this period.—On the 14th the weather was reported to be unsettled over the north and central Bay of Bengal. On the 15th the unsettled conditions in the Bay developed into a depression with the central region lying about 300 miles east of Vizagapatam. On the 16th the depression was found to remain stationary. On the 17th morning it was reported that the depression moved inland and that its central region lay over north-east Hyderabad. By the 18th the depression moved further north-eastwards.

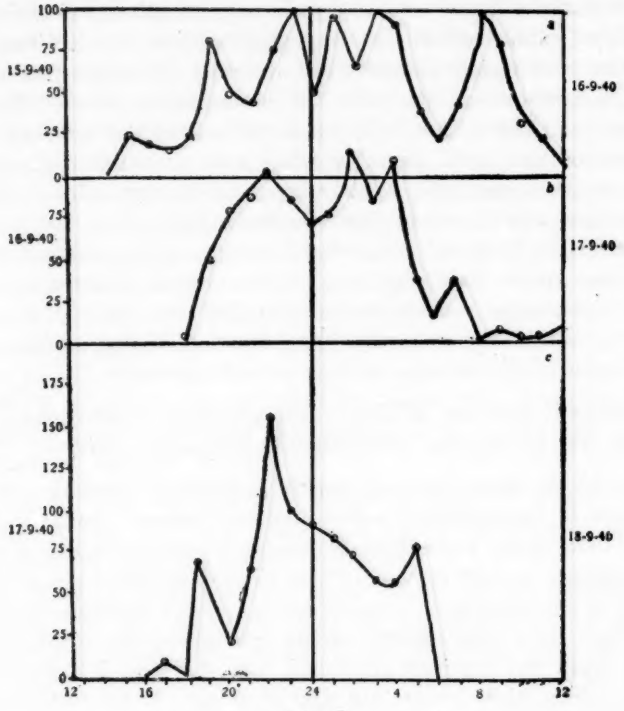

FIG. 5

The group of curves 2*a*, 5*a*, 5*b* and 5*c* seem to bear out the general characteristic variations pointed out in the previous example, *viz.*, (1) continuous night activity during the period in which conditions are reported to be unsettled in the Bay, (2) a simplification of the atmospheric curves as in 5*b*, when a depression is reported to have been formed in the Bay and (3) the occurrence of considerable afternoon activity, terminating in a very large peak of activity as on the night of the 17th but with comparatively very little activity after midnight, coinciding with the reported passing inland of the depression. Indications of this phenomena in 5*b* are not quite so distinct as those observed in the example discussed earlier. Therefore they appear as though they do not indicate clearly the characteristics of the weather mentioned in the *Indian Daily Weather Reports*. A closer examination of the curves in their relation to the weather charts reveals the cause of this apparent discrepancy and supports the conclusions arrived at earlier in an ample measure.

In Fig. 6 are given the weather charts from the 14th to the 18th September 1940. In addition to the isobars, the places from which thunderstorms during the previous twenty-four hours have been reported, are indicated by black dots as explained earlier. For the present we are concerned only with the isobars. The dots will be considered later on.

In spite of the fact that the Weather Reports speak of the formation of a depression on the 15th and the 16th, the simplification of the atmospheric activity curve takes place only on the 17th (Curve 5*b*). On the evening of the 16th, there is activity initiated by sunset and there is considerable activity in the early morning of the 17th between 2 and 4 hours. From 4 hours onwards there is a sudden fall in activity till sunrise. The Curve 5*b*, thus seems to give a clear indication of the formation of the depression.

In this connection it must be remembered that no ship observations were available during this period, and hence the actual position of the isobaric lines is not quite certain. There is a great probability, however, judging from the isobaric lines that could be drawn, that at 17 hours on the evening of the 16th, the isobars have closed in. By 8 hours the next morning the depression is reported to have moved inland. It is significant to note in this connection that definite atmospheric activity is noticed in the early morning of the 17th, in Curve 5*b*. Judging from the evidence at present available, the formation of a well-defined depression and the presence of a well-defined peak of atmospheric activity in the early morning seem to occur together, although it cannot be asserted that the occurrence of atmospheric activity in the early mornings is always connected with the formation of such depressions.

It will therefore not be unreasonable to suggest that the lack of sufficient simplification of the atmospheric curves on the 14th and 15th, in spite of the reported formation of a depression on these days, and the simplification observed on the 17th, may be due to the fact that the depression formed on the 14th was not sufficiently deep and that the area covered by the closed isobar 29″·70 might have been very large ; and that the usual system of close concentric isobars characteristic of a well-formed depression, which Lacoste (1921) considered necessary for the occurrence of definite atmospheric disturbance were absent. The simplification of atmospheric curves as in 5*b* is probably connected with the closing in of the isobars in Fig. 6*d*. However, since the position of the isobaric lines drawn are not quite certain for want of sufficient observational data, the conclusion drawn above is tentative and requires to be confirmed by further investigation.

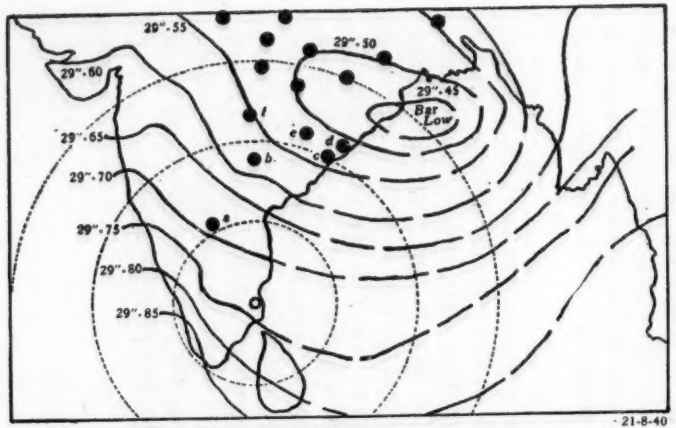

FIG. 4a

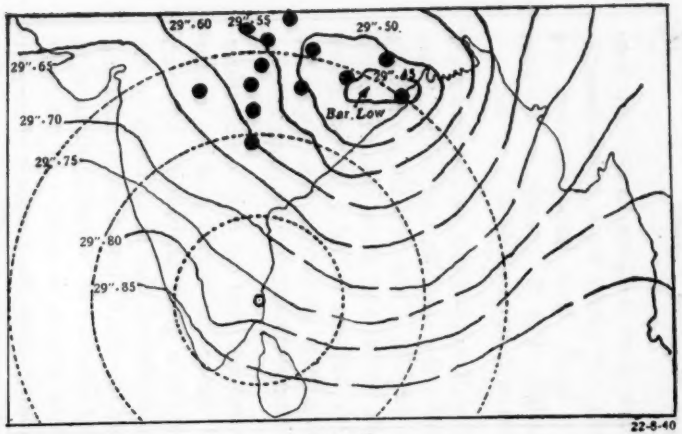

FIG. 4b

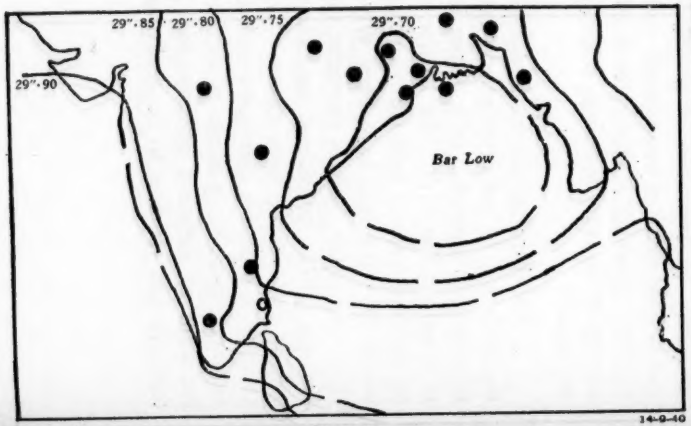

FIG. 6a

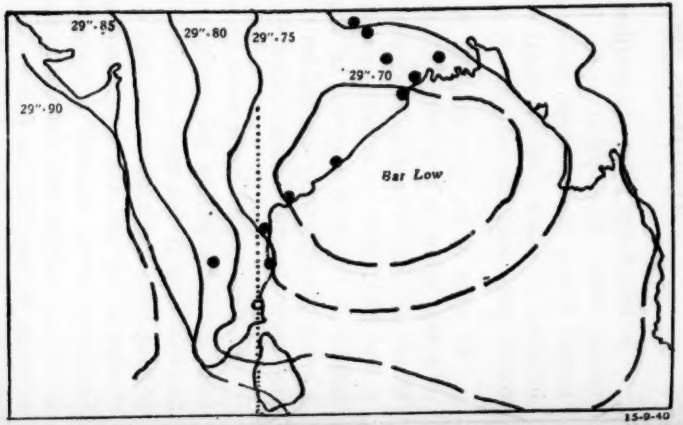

FIG. 6b

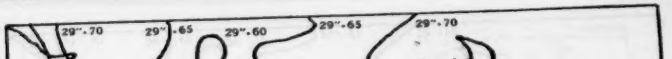

FIG. 6a

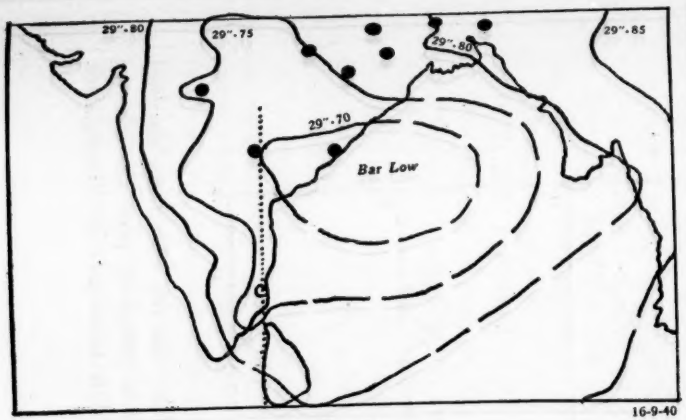

FIG. 6c

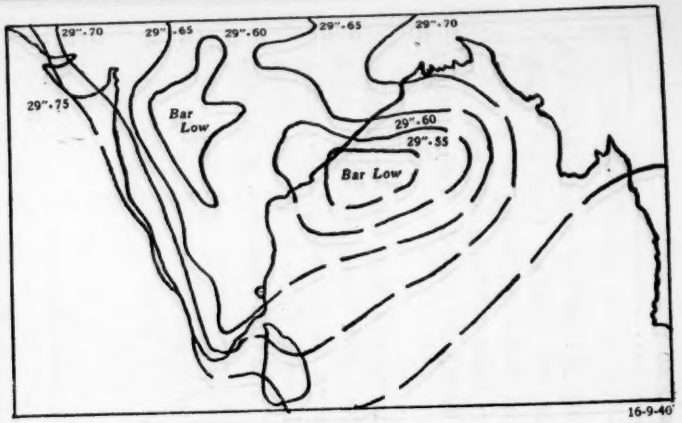

FIG. 6d

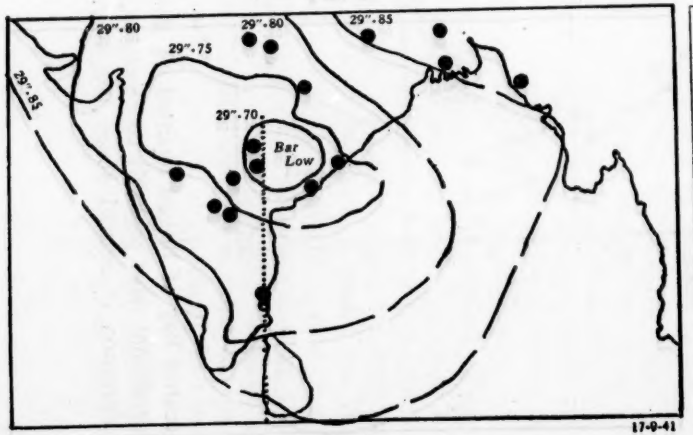

FIG. 6e

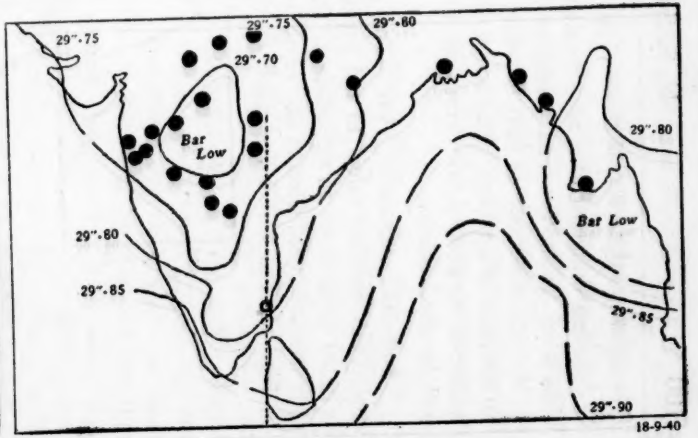

FIG. 6f

The withdrawal of the Monsoon.

Severe thunderstorms are reported from all over the country about the time when the monsoon has withdrawn from the country. These are usually of the type described as heat thunderstorms, and occur mainly on land and in the early afternoons. The occurrence of these storms greatly modify the nature of the atmospheric curves obtained during this period. Two examples will be cited below.

10*th*–20*th September* 1941.—The first example that will be discussed under this heading refers to the period 10th–20th September 1941. As it is found unnecessary to reproduce all the curves corresponding to this period, only five have been selected and reproduced, one as Curve 2*b* and four others in Fig. 7. The first curve refers to the " unsettled conditions in the east

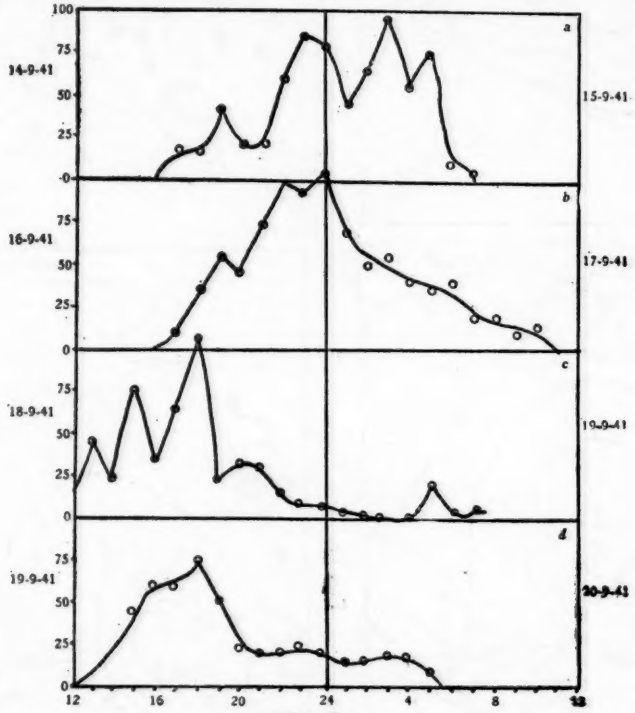

FIG. 7

central Bay of Bengal reported on the 11th. On the 12th these unsettled conditions have probably concentrated into a depression ; on the 13th the depression is centred about 100 miles south south-east of Vizagapatam.

By the 14th the depression passed inland and lay as a diffuse low pressure system over the Deccan. On the 15th it was reported that the trough of low pressure lay over the N. Deccan. By the 19th the trough had moved north-eastwards and there was considerable thunderstorm activity in the south of the Peninsula. On the 20th it is reported that the monsoon had withdrawn from North-West India and that there had been a decrease in rainfall in the southern half of the Peninsula.

Curve 7*c* and 7*d* are of special interest. They refer to the period from 12 noon on the 18th to 12 noon on the 20th September 1941. But it would be interesting to survey the whole period in order to bring out their full significance. Curve 2*b*, as already pointed out, refers to the unsettled conditions which, according to the *Indian Daily Weather Report* of the 11th, prevailed in the east and central Bay of Bengal. The curves for the next three days do not indicate the formation of a depression. These curves have the same characteristic as those exhibited by the curves covering the period from the 14th–16th September 1940. Curve 7*a* shows a slight simplification, and a well-defined activity in the early hours of the morning of the 15th is noticed. Curve 7*b* refers to a period two days later, *i.e.*, 16th–17th, and Curves 7*c* and 7*d* refer to 18th–19th and 19th–20th respectively. The curve for the 17th–18th is omitted as it is very similar to 7*b*.

Curve 7*c* shows three peaks of activity, one at 13 hours, a second at 15 hours and a third at 18 hours. The activity then falls to a low value 25, even by 19 hours and from that time onwards there was no activity at all for the rest of the night and till midday. 7*d* exhibits a peak at 18 hours. But activity on this day starts even at 13 hours. Then it slowly builds up to a maximum value 75 at 18 hours. From that time onwards there is a fall to 20 at 21 hours and after that there is practically no activity.

The presence of large activity in the afternoon and the absence of any activity in the night and the early morning seems to be very peculiar to these two curves. It has been pointed out already that large activity before midnight without any appreciable activity in the early morning is coincident with the fact that the depression has moved inland with the result, that the sources of atmospherics lie mostly on land and their activities are confined to the evening and the early night. Further, it is found that on those days on which evening activity is observed, the activity is usually initiated by sunset. Under the trigger action of sunset, the activity as in Curve 3*b*, rises up to a large value and then diminishes. The rate of decrease is in some cases very rapid and in others gradual.

Curves for the 18th and 19th September 1941 show certain peculiarities. Activity commences soon after midday and continues with varying intensity

till 18 hours. The maximum is attained at this time on each of these days. There seems to be a sudden rise in activity on account of sunset but activity was not initiated by it. It is also significant to note that activity decreases from 18 hours onwards. From (1) 19 hours on the 18th till midday of the 19th and (2) 20 hours on the 19th till noon of the 20th, there was practically no activity.

This peculiar behaviour of Curves 7*c* and 7*d* seems to coincide with the withdrawal of the monsoon. The *Indian Daily Weather Report* of the 19th, reports considerable thunderstorm activity in the south of the Peninsula. It also forecasts that rainfall would decrease in the southern half of the Peninsula, with scattered showers in the north Madras coast and that elsewhere the weather would be dry. On the 20th it says, " Monsoon has withdrawn from North-West India and that there will be a decrease in rainfall in the southern half of the Peninsula. It again forecast thunderstorms in the southern half of the Peninsula and state that elsewhere the weather will be mainly dry." These characteristics of the weather mentioned in the Weather Reports pertaining to the period of the withdrawal of the monsoon are clearly borne out by the Curves 7*c* and 7*d*.

27*th*–29*th September* 1941.—One more example will be cited to bring out the characteristics of the period of the withdrawal of the monsoon. Soon after the withdrawal of the monsoon from North-West India on the 20th, unsettled conditions again prevailed on the 24th and 25th off the Chittagong-Arakan Coast where a depression was expected to form. These conditions are indicated by Curves 2*c* and 2*d*. The unsettled conditions did not however develop into a depression and the monsoon was reported to be withdrawing from Bihar and Chota Nagpur on the 26th. The two curves 8*a* and 8*b* seem to bear out what has been mentioned in discussing the previous example. In 8*a* and 8*b* activity commences at 14 hours and reaches a peak value at 18 hours on the 27th, and at 19 hours on the 28th. But this activity rapidly diminishes on both the days to a very low value by 21 hours. On the 27th activity again shoots up to 110 at 23 hours and then decreases to almost zero value by 4 hours on the 28th. In the afternoon of the 28th, the peak N at 19 hours is almost of the same height as that of L on the previous day. But the peak is very much feebler than M. Zero activity is reached even by midnight and after that although there is some activity it is negligible.

There is a distinct difference between the two curves. Peaks L and M are quite distinct. As pointed out earlier this indicates that the atmospherics recorded originate from two independent sources of activity. The activities

rose to their peaks in a short time and also quickly died down. On the 28th however, it took 6 hours for the peak N to attain indicating that the activity is over a wider region. But it died down in 2 hours.

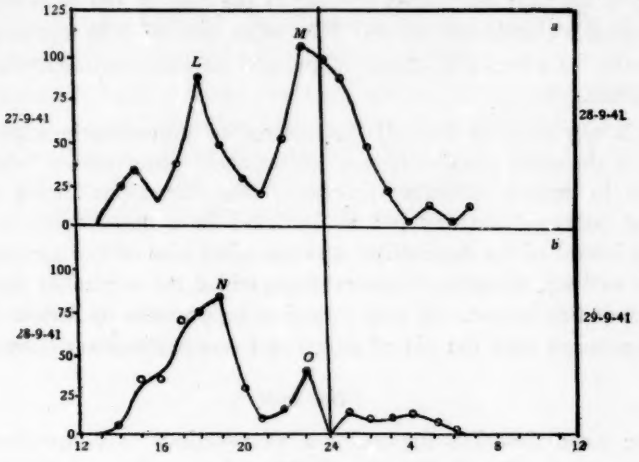

FIG. 8

It is therefore quite clear from the two examples discussed above, that the period corresponding to the withdrawal of the monsoon is associated with activity in the afternoons. The afternoon activity is due to what may be described as heat thunderstorms. This is not accompanied by any activity in the later part of the night and the early hours of the morning.

Location of meteorological depressions with the aid of atmospherics.

The study of atmospherics in the present investigation has been carried out with a horizonatl aerial, which does not exhibit any directional properties. If, however, a directional aerial be employed, with its aid it will be possible to obtain the direction in which the centres of activity lie. If observations from two interconnected stations be simultaneously carried out, the actual position of the sources can also be determined.

Even if only one station is making the observations valuable information can be obtained with the aid of directional equipment. The nature of the information that may expected to be obtained and its value for detecting the formation, location and movement of a depression may be briefly explained with reference to Figs. 6*b*, 6*c*, 6*d*, 6*e* and 6*f*. In these figures is drawn a dotted line to indicate the north-south direction through the observation post at Annamalainagar.

It will be seen from the above-mentioned figures that the regions of atmospheric activity lie almost entirely to the east of the N–S line in Figs. 6*b*, and 6*c*. In 6*e* they are equally distributed to the east and west of this line. In Fig. 6*f* they are distributed entirely to the west of the N–S line. During this period a depression formed 300 miles east of Vizagapatam, remained stationary for a day, and moved inland and travelled north-eastwards through Hyderabad.

It is very unlikely that all the centres of atmospheric activity will be active at the same time. Hence directional observations will indicate maxima in certain definite directions, the directions being different at different parts of the day but all included in a narrow sector. With the moving inland of the depression, and therefore also of the regions of atmospheric activity, directional observations will show a gradual anti-clockwise rotation of this sector. It may therefore be possible to follow the motion of a depression with the aid of directional observations on Atmospherics.

Summary

The paper describes the results of observations on Atmospherics during the monsoon months of August and September in the years 1940 and 1941. Atmospherics were recorded for about two minutes at intervals of one hour throughout the day and every day throughout the period under discussion. From the records, ' atmospheric activity ' at various parts of each day has been evaluated and its variation is represented graphically.

During the period under discussion, depressions usually form in the Bay of Bengal and gradually move either west or north-west into Central India and disappear from North-West India. The atmospheric activity curves are discussed in relation to the meteorological data collected at a large number of places distributed throughout India and published daily by the Indian Meteorological Deparatment as the *Indian Daily Weather Report*.

The paper discusses the nature of Atmospherics accompanying (1) the unsettled conditions which precede the formation of depressions in the Bay of Bengal, (2) the formation of depressions in the Bay of Bengal, (3) the movement of the depression inland, and (4) the withdrawal of the monsoon.

It is pointed out that unsettled conditions are associated with continuous activity throughout the night. The formation of a depression is accompanied by large atmospheric activity in the early hours of the morning, leading to a well-defined peak of activity. The movement of the depression inland immediately brings about increased early night activity initiated by sunset, while the activity in the early hours of the morning disappears. With the depression

moving still further inland, the scale of the phenomena becomes very much reduced. The withdrawal of the monsoon is accompanied by heat thunderstorms which give rise to atmospheric activity in the afternoons and the evenings. This is not, however, accompanied by any activity in the later part of the night and in the early hours of the morning.

It is further pointed out that valuable information regarding the formation, location, and movement of depressions can be obtained with the aid of directioanl receivers. Attempts are being made to simplify the recording equipment and to test the usefulness of this method. The results of further investigations will be communicated in due course.

My thanks are due to Dr. S. Ramachandra Rao, D.Sc. (London), Professor of Physics, for helpful suggestions and criticism. I thank also Mr. M. V. Subrahmanyam, Mechanic, for his valuable help in connection with the recording of atmospherics.

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